Surface Curvature Effects on Hypersonic Boundary Layers," *AIAA Journal*, Vol. 10, No. 12, Dec. 1972, pp. 1557–1558.

[5] Harris, J. E., "Numerical Solutions of the Equations for Compressible Laminar, Transitional and Turbulent Boundary Layers and Comparisons with Experimental Data," TR R-368, Aug. 1971, NASA.

[6] Werle, M. J., Polak, A., and Bertke, S. D., "Supersonic Boundary Layer Separations and Reattachment—Finite Difference Solutions," AFL 72-12-1, Jan. 1973, University of Cincinnati, Cincinnati, Ohio.

[7] Polak, A., "Interacting Supersonic Turbulent Boundary Layer over a Two-dimensional Protuberance," NOLTR 74-16, Jan. 1974, Naval Ordnance Laboratory, White Oak, Md.

Flexure of Elastically Supported Axisymmetric Shells

S. T. GULATI*
Corning Glass Works, Corning, N.Y.

AND

M. C. DOKMECI†
Deniz Harp Okulu, Istanbul, Turkey

Introduction

THE present Note is concerned with the flexure of axisymmetric cylindrical shells with elastically supported edges. In particular, the effects of support parameters, namely support width and support stiffness, on the stress distribution throughout the shell are examined using Reissner-Naghdi shell theory.[1,2] In practice, one seldom encounters either rigidly clamped or simply supported edges. Indeed, in the case of shells made of brittle materials, it is undesirable and impractical to clamp the edges because of the presence of microcracks at the cut edges. On the other hand if the edges are simply supported, the maximum value of axial bending moment, for example, in a short uniformly loaded circular cylindrical shell is of substantial magnitude. Thus, by proper choice of the support parameters it should be possible to reduce the absolute value of maximum bending moment and to ascertain its location away from the edges. The analogous problem for rectangular plates has been treated by Gulati and Buehl,[3] while that for axisymmetric plates was examined by Reismann[4] and more recently by Gulati.[5]

Using the basic equations of Reissner-Naghdi shell theory, the general solution for the deflection and stress resultants is obtained for the region away from the support and the support region. The details of the solution are omitted; however, the results based on this solution are compared with those corresponding to axisymmetric cylindrical shells with simply supported or clamped edges. The effect of length/diam ratio and thickness/diam ratio on the stress resultants is examined for different values of support parameters. It is shown that many beneficial effects derive from elastic supports.

Analysis

For the circular cylindrical shell, Fig. 1, of midsurface radius a, thickness h, and length $2L$ supported elastically on gaskets of spring modulus K and width $(L-c)$ and subjected to an internal pressure p, the basic equations of Reissner-Naghdi theory as modified by Essenburg[6] result in the following differential equations for axial moment M_x in the inner and support regions respectively:

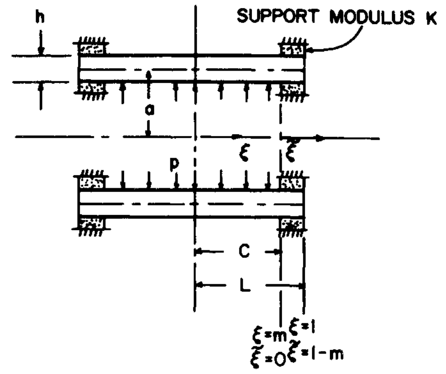

Fig. 1 Shell geometry and coordinate system.

$$\phi^{iv}-2(\omega^2-\zeta^2)\eta^2\phi''+4\eta^4\phi = -(\nu/\lambda^2)(p/E)[1-(\gamma/2\lambda)],\quad 0\leqq\xi\leqq m \tag{1}$$
$$\bar{\phi}^{iv}-2(\bar{\omega}^2-\bar{\zeta}^2)\bar{\eta}^2\bar{\phi}''+4\bar{\eta}^4\bar{\phi}=0,\quad 0\leqq\bar{\xi}\leqq 1-m$$

where

$$\begin{aligned}
&\phi = M_x/EL^2 = \bar{M}_x && m = c/L\\
&\bar{\phi} = \tilde{M}_x/EL^2 = \bar{\bar{M}}_x && K' = K(L-c)\\
&\gamma = h/L && \xi = x/L\\
&\lambda = a/L && \bar{\xi} = \xi - m\\
&\omega^2,\bar{\omega}^2 = 1+\rho^2(\eta^2/\mu^2), && 1+\bar{\rho}^2(\bar{\eta}^2/\mu^2)\\
&\zeta^2,\bar{\zeta}^2 = 1-\rho^2(\eta^2/\mu^2), && 1-\bar{\rho}^2(\bar{\eta}^2/\mu^2)\\
&\rho^2,\bar{\rho}^2 = 1-\frac{\nu\gamma^2\mu^2}{12(1-\nu^2)}, && 1-\frac{\nu\gamma^2\mu^2}{12(1-\nu^2)\Omega^4}\\
&\eta^4,\bar{\eta}^4 = \frac{3(1-\nu^2)}{\gamma^2\lambda^2}, && \frac{3(1-\nu^2)\Omega^4}{\gamma^2\lambda^2}\\
&\mu^2 = \frac{5(1-\nu)}{\gamma^2}, && \Omega^4 = 1+\frac{2K'\lambda^2}{E\gamma(1-m)}
\end{aligned}$$

and E and ν are the elastic modulus and Poisson's ratio of the shell material. The solution of Eqs. (1) is given by

$$\phi(\xi) = M_1\cosh e\xi\cos b\xi + M_2\sinh e\xi\sin b\xi - \frac{\nu\gamma^2}{12(1-\nu^2)}\left(\frac{p}{E}\right)\left(1-\frac{\gamma}{2\lambda}\right) \tag{2}$$
$$\bar{\phi}(\bar{\xi}) = \tilde{M}_1\cosh\bar{e}\bar{\xi}\cos\bar{b}\bar{\xi}+\tilde{M}_2\sinh\bar{e}\bar{\xi}\sin\bar{b}\bar{\xi}+\tilde{M}_3\cosh\bar{e}\bar{\xi}\sin\bar{b}\bar{\xi}+\tilde{M}_4\sinh\bar{e}\bar{\xi}\cos\bar{b}\bar{\xi}$$

in which

$$e,\bar{e} = (2)^{1/2}\eta\cos(\alpha/2),\ (2)^{1/2}\bar{\eta}\cos(\bar{\alpha}/2)$$
$$b,\bar{b} = (2)^{1/2}\eta\sin(\alpha/2),\ (2)^{1/2}\bar{\eta}\sin(\bar{\alpha}/2)$$
$$\alpha,\bar{\alpha} = \cos^{-1}\left(\frac{\eta^2}{\mu^2}\rho^2\right),\ \cos^{-1}\left(\frac{\bar{\eta}^2}{\mu^2}\bar{\rho}^2\right)$$

The six constants of integration M_1, M_2, $\tilde{M}_1,\ldots,\tilde{M}_4$ are readily found by satisfying four continuity conditions at $\xi = m$ and two boundary conditions at $\bar{\xi} = 1-m$. The classical theory (Love's First Approximation) solution follows from Eqs. (2) by examining the limit $\mu\to\infty$.

Results and Discussion

The abovementioned solution will now be illustrated by way of an example. Since we are primarily interested in flexure and the effects of support parameters on the stress resultants, we consider the problem shown in Fig. 1 with $p = 10^{-5}$ E. The numerical computations were carried out on the computer and the results are shown in Figs. 2–5.

The effect of support width on axial moment distribution for a given value of support modulus (10^3 psi) is shown in Fig. 2 where the predictions of both shear deformation theory and classical theory are plotted for $(L/a) = 1.0$ and $(h/a) = 0.3$. As would be

Received December 26, 1973; revision received May 2, 1974.

Index categories: Structural Static Analysis; Structural Design, Optimal.

* Physical Properties Research Department, Research and Development Laboratories; also Adjunct Associate Professor, Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, N.Y.

† Naval Officer, Turkish Naval Academy.

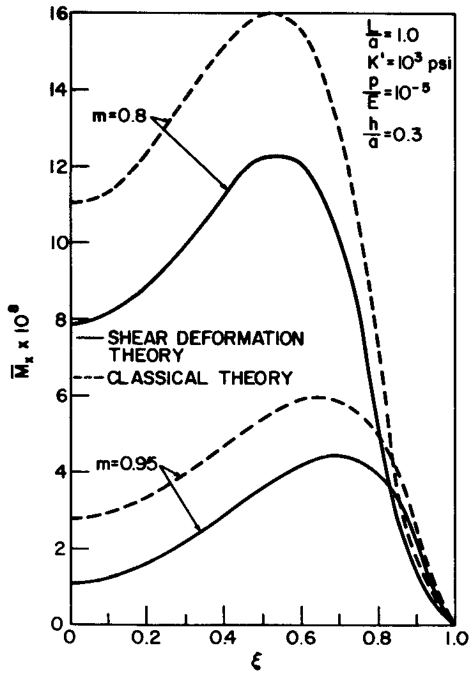

Fig. 2 Axial moment distribution for two different support widths; $K' = 10^3$ psi, $L/a = 1.0$, $h/a = 0.3$.

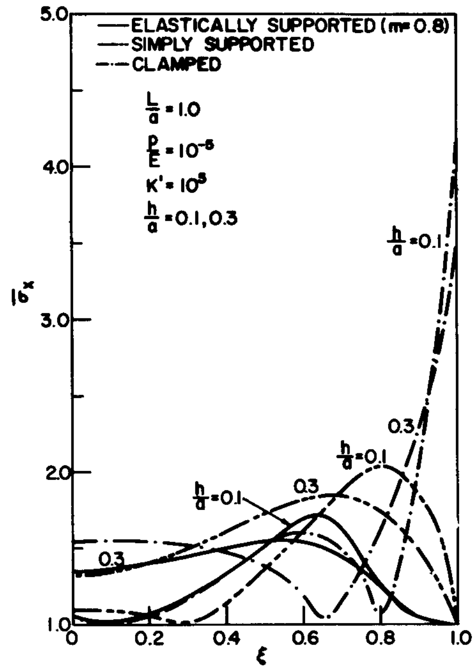

Fig. 4 Distribution of resultant axial stress for $L/a = 1.0$ and different values of h/a; $K' = 10^5$ psi, $m = 0.8$.

expected, a greater support width, i.e., smaller value of m, results in a larger value of maximum axial moment. There is a two-fold increase in the maximum axial moment when m is reduced from 0.95 to 0.8. It is also clear from this figure that the classical theory predicts 35% higher value for $\bar{M}_x^{\max}$ for $m = 0.95$.

Next we compare the axial moment distribution for two different values of K' with that of simply-supported and clamped shells, Fig. 3. For the K' values chosen, the support conditions are similar to those of a simply supported shell but the magnitude of axial moment is lower by a substantial amount. Thus with a finite value of support modulus, $K' = 10^3$ psi, axial moment can be kept as low as 20% of that in a simply supported shell.

The distribution of resultant axial stress (in terms of $\bar{\sigma}_x = 2h\sigma_x/pa$) for two values of h/a and L/a is shown in Figs. 4 and 5. For comparison purposes this distribution for a simply-supported and clamped shell is also shown. It should be noted that the contribution of flexural stress to the resultant stress is significant, approaching 50% value. This contribution for a given value of K' decreases as $m \to 1$. It is also clear that the maximum resultant stress under simply supported and clamped edge assumption is considerably larger and may result in over-designing the shell thickness. An examination of the resultant

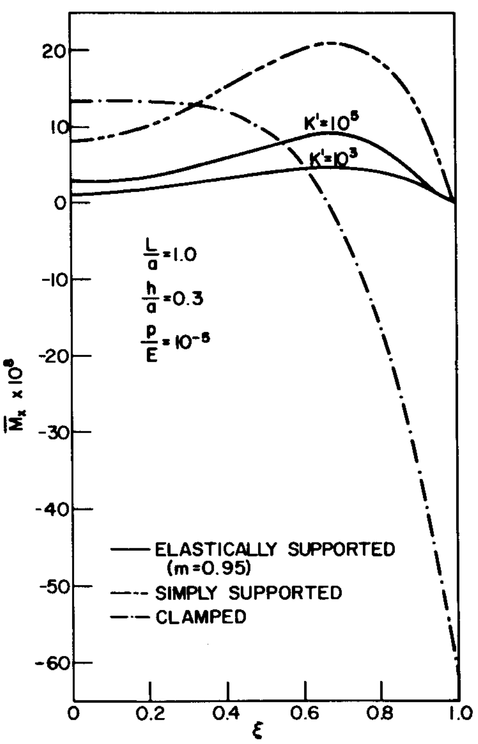

Fig. 3 Axial moment distribution for simply-supported, clamped and elastically supported shells; $K' = 10^3$ and 10^5 psi, $m = 0.95$, $L/a = 1.0$, $h/a = 0.3$.

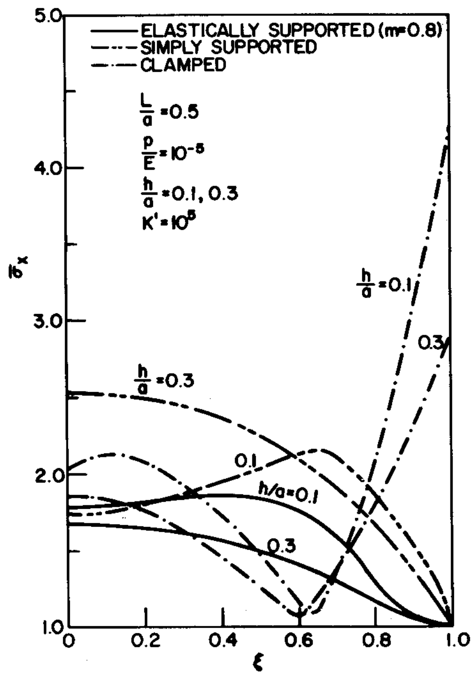

Fig. 5 Distribution of resultant axial stress for $L/a = 0.5$ and different values of h/a; $K' = 10^5$ psi, $m = 0.8$.

circumferential stress indicates that the flexural stress contribution amounts to a maximum value of 15% of the membrane stress in that direction and is therefore not plotted. Klosner and Levine,[7] in comparing the elasticity and shell theory solutions, have found that for very short shells subjected to discontinuous transverse normal loads, even higher-order shell theories such as the one used here can give inaccurate results for the stresses. For this reason, the minimum value of L/a has been restricted to 0.5 in Fig. 5. Moreover, since the loads considered here are continuous, the stresses computed by Reissner-Naghdi theory should be fairly accurate.

Conclusions

Using Reissner-Naghdi shell theory, it is found that substantial reduction in flexural stresses can be effected by supporting the edges elastically. Moreover, for support modulus values as large as 10^5 psi the edge moment is almost zero and the maximum value of bending moment occurs between the edge and mid-length of the shell. It is also found that the support width should be kept as small as practical in order to keep the magnitude of flexural stresses low.

Finally, a comparison of classical vs shear deformation theory indicates that although for thin shells the two theories predict almost identical moment and shear resultant, the classical theory overestimates stresses by about 35% for short, thick shells.

References

[1] Reissner, E., "Stress-Strain Relations in the Theory of Thin Elastic Shells," *Journal of Mathematical Physics*, Vol. 31, 1952, pp. 109–119.

[2] Naghdi, P. M., "On the Theory of Thin Elastic Shells," *Quarterly of Applied Mathematics*, Vol. 14, No. 4, Jan. 1957, pp. 369–380.

[3] Gulati, S. T. and Buehl, W. M., "Rectangular Plates with Elastic Edge Supports," *Transactions of the ASCE, Journal of Engineering Mechanics Division*, Vol. 96, No. EM4, Aug. 1970, pp. 521–528.

[4] Reismann, H., "Bending and Buckling of an Elastically Restrained Circular Plate," *Journal of Applied Mechanics*, Vol. 19, No. 2, June 1952, pp. 167–172.

[5] Gulati, S. T., "Effect of Support Elasticity on the Bending of Axisymmetric Plates," *AIAA Journal*, Vol. 8, No. 9, Sept. 1970, pp. 1635–1638.

[6] Essenburg, F., "Thermal Elastic Cylindrical Shells with Surface Constraints," *Proceedings of the IASS Symposium on Nonclassical Shell Problems*, Warsaw, 1963, pp. 100–115.

[7] Klosner, J. M. and Levine, H. S., "Further Comparison of Elasticity and Shell Theory Solutions," *AIAA Journal*, Vol. 4, No. 3, March 1966, pp. 467–480.

Kinetic Factors in Heterogeneous Combustion of Cross-Linked Polymers

S. K. BAIJAL* AND A. N. SRIRAMULU†
Indian School of Mines, Dhanbad, India

Nomenclature

A = Arrhenius frequency factor, sec^{-1}
B' = B, thermochemical mass transfer number
C_p = specific heat, cal/g°C
D = instantaneous duct diameter, cm
E = energy of activation in thermal degradation, Kcal/mole
G_o = initial oxidizer rate, g/cm^2 sec
G = total mass flow rate, g/cm^2 sec
K = thermal conductivity, cal/cm sec°C
L = length of the grain, cm
$\dot{m}$ = mass regression rate, g/cm^2 sec
$\dot{r}$ = linear regression rate, cm/sec
Re_x = Reynolds number based on x
T = temperature, °K
ρ = density, g/cm^3
α = thermal diffusivity, cm^2/sec
$\left(\frac{\bar{\rho}}{\rho_e}\right)$ = semiempirical variable property correlation factor

Subscripts
o = initial
s = surface
f = fuel

Introduction

THE heat and mass transfer within the turbulent boundary layers has been investigated by Marxman et al.[1–3] in heterogeneous combustion of linear polymers. The regression rate of similar polymers has been correlated by Houser[4] and Rabinovitch[5] with kinetics of degradation in solid phase. In all these investigations the polymers are derived from vinyl monomers, where considerable mechanical strength is reduced with time. In this investigation more precise data have been obtained in heterogeneous combustion of cross-linked polyesters to study the effect of kinetic factors in solid phase.

Experimental

A. Preparation of Polymers

Four polyesters were prepared by reacting propylene glycol with a suitable ratio of maleic anhydride and phthalic anhydride. The composition of linear polyester is varied by varying the ratio of maleic and phthalic anhydride. The linear polyester resin is crosslinked with styrene. The molar ratio of maleic anhydride to styrene is selected either 1:1 or 1:2. The molar ratio of maleic anhydride, phthalic anhydride, and styrene in polymer is shown as M:P:S, respectively, and the physical properties are tabulated in Table 1.

B. Degradation Studies

The experimental procedure of Rastogi and Gupta[6] was adopted for finding the degradation constant. The least-square technique was used to evaluate the degradation rate constant, Arrhenius factor, and energy of activation. The experimental data were processed in IBM-360-44 computer and the mean values of kinetic factors are given in Table 1.

C. Regression Rate and Surface Temperature

The cylindrical grains (o.d. 5 cm, length 8.5 cm) having an L/D_o ratio around 10 were used for combustion in a tubular burner of similar dimensions. The diameter of circular inlet for gases is around 0.7 cm and flow of gases was controlled by a three-way valve in between flowmeter and burner. The heterogeneous reaction was started by igniting the fuse wire electrically by Parr Ignition Unit. The commercial oxygen was used for combustion and nitrogen for extinguishing the flame. The exit end of burner was open to atmosphere. The thermocouple, prepared from copper-constantan wires,‡ was inserted in the center of the grain and its leads were connected to a strip-chart recorder.§ The typical temperature-time traces until the thermocouple is in solid phase are shown in Fig. 1. The average linear regression rate was calculated by measuring the mean radius before and after the combustion for a period of time. The mass regression rate was calculated by finding loss in weight during combustion. The time of combustion was usually selected between 20–60 sec to assure the steady-state conditions are attained at the end of the run. If heat of pyrolysis of polymer is

Received September 27, 1972; revision received May 16, 1974. The authors would like to thank G. S. Marwaha, Director, Indian School of Mines, for providing the necessary facilities.

Index categories: Combustion in Heterogeneous Media; Fuels and Propellants, Properties of.

* Assistant Professor, Department of Petroleum Technology.
† Lecturer, Department of Petroleum Technology.

‡ 0.001 in. Omega Engineering Inc., Stamford, Conn.
§ Honeywell: 0–10 mv.